increased in thickness with increasing aspect ratio. This was particularly noticeable in the profiles taken with $\alpha = 45°$. Inflexion points in the temperature profiles taken inside cavities with $\alpha = 45°$, and $a/b = 1$ and 1.48 respectively, attested to the presence of a counterclockwise-recirculating motion in the stratified flow region within the cavity. This recirculation was confirmed by the flow visualization results.

The present note is imperfect in the sense that detailed quantitative bounds are not provided for the dimensionless parameters governing the behavior of pulsating thermally-driven cavity flow. For this, continued experimentation is necessary and is currently underway. Nevertheless, the information advanced herein is timely and points to a phenomenon which has not previously been observed and may be present in the work of others.

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TEMPERATURE DISTRIBUTION IN A LARGE CIRCULAR PLATE HEATED BY A DISK HEAT SOURCE

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NOMENCLATURE

- b, thickness of circular plate;
- $E(\ )$, complete elliptic integral of the second kind;
- $F(\ )$, hypergeometric function;
- $J_i(\ )$, Bessel function of the first kind;
- $K(\ )$, complete elliptic integral of the first kind;
- k, thermal conductivity;
- q, heat flux;
- r, radial coordinate;
- r_0, radius of heated area;
- T, temperature;
- T_w, temperature of coolant;
- z, axial coordinate.

Greek symbols

- θ, temperature rise, $T - T_w$;
- ∇^2, Laplace operator;
- $\xi(n)$, Riemann's zeta function;
- $\Gamma(m)$, gamma function.

INTRODUCTION

THE HEAT transfer analysis of disk-shaped heating of a large solid plate is commonly required in spot welding, fire safety, the anode and cathode of an MPD arc, the cooling of electronic equipment and for electric circuit breakers. Thomas [1] gave an exact solution in terms of tabulated functions for a semi-infinite body heated by a constant heat flux in the region $0 \leqslant r \leqslant r_0$ at $z = 0$, with the other surface insulated. The purpose of the present note is to consider the isotherm at $z = b$, and to determine the effect of plate thickness on the temperature distribution. The solution derived is for the steady state heat conduction problem. This solution is useful in the development of a new series solution [2, 3] for transient temperature in a solid where the solution at the surface takes advantage of a known steady state solution.

ANALYSIS

The circular plate is considered to be isotropic and homogeneous. The geometry and coordinate are shown in Fig.

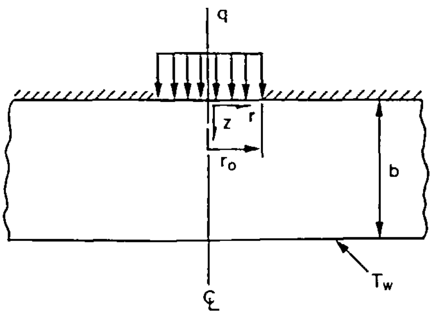

FIG. 1. A large circular plate heated over a disk-shaped centred at $r = 0$ and $z = 0$, insulated elsewhere at $z = 0$. The opposite surface, $z = b$, is isothermal.

1. A mathematical statement for steady state conduction problem is the solution of Laplace's equations

$$\nabla^2\theta(r,z) = 0 \tag{1}$$

with the boundary conditions

$$\begin{aligned} -k\frac{\partial\theta(r,0)}{\partial z} &= q, \quad 0 \leqslant r \leqslant r_0 \\ &= 0, \quad r > r_0 \end{aligned} \tag{2}$$

and

$$\theta(r,b) = 0. \tag{3}$$

The infinite Hankel transformation [4] is given by

$$\bar{\theta}(p,z) = \int_0^\infty rJ_0(pr)\theta(r,z)\,\mathrm{d}r, \tag{4}$$

$$\theta(r,z) = \int_0^\infty p\bar{\theta}(p,z)J_0(pr)\,\mathrm{d}p. \tag{5}$$

Applying the Hankel transformation (4) to equations (1)–(3) and inverting it by using equation (5), the general solution of equations (1)–(3) is

$$\theta(r,z) = \int_0^\infty pa\{\exp(-pz)-\exp[p(z-2b)]\}J_0(pr)\,\mathrm{d}p \tag{6}$$

where a is constant.

From equation (2), the boundary conditions become

$$\begin{aligned} \int_0^\infty f(p)J_0(pr)\,\mathrm{d}p &= \frac{q}{k}, \quad 0 \leqslant r \leqslant r_0 \\ &= 0, \quad r > r_0 \end{aligned} \tag{7a}$$

where

$$f(p) = ap^2[1+\exp(-2bp)]. \tag{7b}$$

The arbitrary function $f(p)$ is determined from an expression given by Watson [5] and the solution of the heat conduction equation is

$$\theta(r,z) = \frac{qr_0}{k}\int_0^\infty \frac{\{\exp(-pz)-\exp[p(z-2b)]\}}{p[1+\exp(-2bp)]} \times J_0(pr)J_1(pr_0)\,\mathrm{d}p. \tag{8}$$

This expression is valid for all r and z values equal to and greater than zero. To evaluate the infinite integral, it is convenient to express in terms of hyperbolic functions and then in a power series [5] of $(1/2sb)$.

Using the result of Whittaker and Watson [6] for Riemann's zeta function,

$$\begin{aligned} \sum_{s=1}^{\infty}\frac{(-1)^{s-1}}{s^{2m+1}} &= (1-2^{-2m})\xi(2m+1), \quad m > 0 \\ &= \ln 2, \quad m = 0 \end{aligned} \tag{9}$$

where $\xi(\ )$ is Riemann's zeta function and is tabulated in ref. [7].

A location of particular interest is at the surface, $z = 0$, where the temperature distribution contains an expression for the infinite plate thickness ($b/r_0 \to \infty$) plus a correction term due to finite thickness. Thus the temperature distribution can be written as

$$\theta(r,0) = \frac{2qr_0}{\pi k}E\left(\frac{r}{r_0}\right) \quad \text{for} \quad 0 \leqslant r \leqslant r_0 \tag{10a}$$

and for $r > r_0$

$$\theta(r,0) = \frac{qr_0}{k}\left\{\frac{2r}{\pi r_0}\left[E\left(\frac{r_0}{r}\right)-\left(1-\frac{r_0^2}{r^2}\right)K\left(\frac{r_0}{r}\right)\right]\right\}, \tag{10b}$$

and a correction term (due to finite thickness)

$$-\frac{2qr_0}{k}\left\{\frac{r_0}{4b}\left[1+\frac{r^2}{4b^2}\xi(2)\right]^{-1/2}\ln 2 + \sum_{m=1}^{\infty}\frac{(-1)^m(1-2^{-2m})\xi(2m+1)\Gamma(2m+1)}{m!\Gamma(m+2)} \times \left(\frac{r_0}{4b}\right)^{2m+1} {}_2F_1\left[m+\tfrac{1}{2}, m+1; 1; -\frac{r^2}{4b^2}\xi(2)\right]\right\}, \tag{11}$$

which is common to both equations (10a) and (10b) where ${}_2F_1(\ )$ is the hypergeometric function and is defined in refs. [6, 8]. The functions $K(\ )$ and $E(\ )$ are the complete elliptic integrals of the first and second kinds,

$$K(\varphi) = \int_0^{\pi/2}[1-\varphi^2\sin^2\theta]^{-1/2}\,\mathrm{d}\theta, \tag{12a}$$

$$E(\varphi) = \int_0^{\pi/2}[1-\varphi^2\sin^2\theta]^{1/2}\,\mathrm{d}\theta. \tag{12b}$$

These functions are tabulated in refs. [7, 8].

The convergence of the infinite series is now examined. It is mentioned by Watson [5] that the criterion for convergence of the series is given by $|r_0/2b| < 1$. This limit is satisfied in many practical cases. These exact solutions are very efficient and need just a few terms to obtain a satisfactory solution of the temperature distribution.

If the higher order terms are neglected in the correction term (11), it can be seen that the temperature depression due to a finite thickness is approximately proportional to the square of the heated spot radius and inversely proportional to the thickness of the plate in the region $0 \leqslant r \leqslant r_0$. The temperature distribution at the surface is shown in Fig. 2, where the dimensionless temperature $\theta k/qr_0$ is plotted vs dimensionless radial position r/r_0 for various values of the dimensionless plate thickness b/r_0 at $z = 0$.

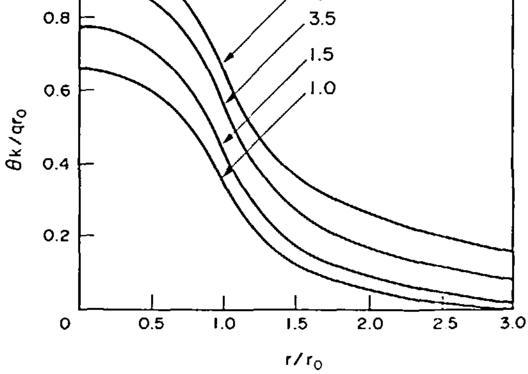

FIG. 2. Dimensionless temperature distribution for various values of b/r_0 at $z = 0$.

The mean temperature, $\bar{\theta}(\bar{r},0)$ over the disk-shaped heating region, $0 \leqslant r \leqslant r_0$ is of interest for obtaining contact conductance and other applications. To calculate the mean temperature, one can multiply $\theta(r,0)$ by $2\pi r\,dr$, integrate from $r=0$ to r_0 and divide by $2\pi r_0$.

The results are

$$\bar{\theta}(\bar{r},0)=\frac{qr_0}{k}\left[\frac{8}{3\pi}-4\sum_{m=0}^{\infty}\frac{(-1)^m(1-2^{-2m})\xi(2m+1)\Gamma(2m+1)\Gamma(2m+3)}{m!\,\Gamma(m+2)\Gamma(m+2)\Gamma(m+3)}\left(\frac{r_0}{4b}\right)^{2m+1}\right]. \quad (13)$$

For a semi-infinite solid, the last terms on the RHS of equation (13) vanish and the maximum temperature at the centreline of the plate $r=0$, is qr_0/k and the mean temperature over the area $0 \leqslant r \leqslant r_0$ is $8qr_0/3\pi k$, as obtained by Thomas.

The temperature distribution at $0 < z \leqslant b$ can be expressed in a series form. Equation (8) becomes

$$\theta(r,z)=\frac{qr_0}{k}\sum_{s=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^s(-1)^m\Gamma(2m+1)}{m!\,\Gamma(m+2)}\left(\frac{r_0}{4b}\right)^{2m+1}\times\left[a_1^{-2m-1}{}_2F_1\left(m+\tfrac{1}{2},m+1;1;-\frac{r^2}{4a_1^2b^2}\right)-b_1^{-2m-1}{}_2F_1\times\left(m+\tfrac{1}{2},m+1;1;-\frac{r^2}{4b_1^2b^2}\right)\right] \quad (14)$$

where

$$a_1=[s+(z/2b)]$$

and

$$b_1=[s+1-(z/2b)].$$

The temperature field $\theta k/qr_0$ is shown in Fig. 3 for the nondimensional plate thickness $b/r_0 = 3.5$.

CONCLUSIONS

An exact steady state solution is developed for a large circular plate heated with a disk source centred and insulated elsewhere, whereas the opposite surface is considered isothermal. The solutions are written in terms of power series which converge very fast. For this reason their series solution required only ten terms for the evaluation of the temperature distribution.

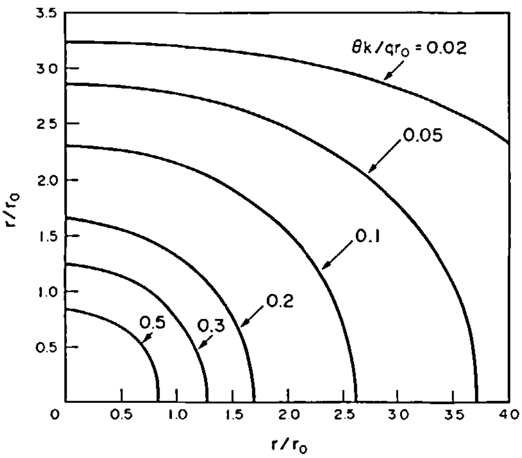

FIG. 3. Dimensionless temperature field for $b/r_0 = 3.5$.

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DROPLETS ON IMPACT WITH A SOLID SURFACE

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NOMENCLATURE

- a, thermal diffusivity;
- b, thickness of the liquid layer;
- c, specific heat;
- D, droplet diameter;
- Δh, latent heat of fusion;
- k, parameter, $6\varepsilon^2U(\rho/\rho')(\varepsilon/Pe)^{1/2}$;
- Pe, Péclet number, wD/a;
- Re, Reynolds number, $\rho' wD/\mu$;
- R, radius of the splat, $R(t)$;
- T, dimensionless temperature, $(T'_s - T')/c\Delta h$;
- T', temperature;
- T'_s, temperature of freezing;
- T'_0, temperature of the solid surface;
- t, dimensionless time, $wt'/R(0)$;
- t', time;
- U, freezing constant;
- We, Weber number, $\rho' w^2 D/\sigma$;